ON BLACK HOLE SOLUTIONS IN MODEL WITH ANISOTROPIC FLUID

H. Dehnen [1],

Universität Konstanz, Fakultät für Physik, Fach M 568, D-78457, Konstanz

V.D. Ivashchuk[2] **and V.N. Melnikov** [3]

Center for Gravitation and Fundamental Metrology, VNIIMS, 3/1 M. Ulyanovoy Str., Moscow 119313, Russia and

Institute of Gravitation and Cosmology, PFUR, Michlukho-Maklaya Str. 6, Moscow 117198, Russia

Abstract

A family of spherically symmetric solutions in the model with 1-component anisotropic fluid is considered. The metric of the solution depends on a parameter $q > 0$ relating radial pressure and the density and contains $n-1$ parameters corresponding to Ricci-flat "internal space" metrics. For $q = 1$ and certain equations of state ($p_i = \pm\rho$) the metric coincides with the metric of black brane solutions in the model with antisymmetric form. A family of black hole solutions corresponding to natural numbers $q = 1, 2, \ldots$ is singled out. Certain examples of solutions (e.g. containing for $q = 1$ Reissner-Nordström, $M2$ and $M5$ black brane metrics) are considered. The post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric are calculated.

[1] Heinz.Dehnen@uni-konstanz.de

[2] ivas@rgs.phys.msu.su

[3] melnikov@rgs.phys.msu.su

1 Introduction

Currently, there is a certain interest to p-brane solutions with horizon (see, for example, [1] and references therein) defined on product manifolds $\mathbb{R} \times M_0 \times \ldots \times M_n$. P-brane solutions (e.g. black brane ones) usually appear in the models with antisymmetric forms and scalar fields (see also [4]-[14]). Cosmological and spherically symmetric solutions with p-branes are usually obtained by the reduction of the field equations to the Lagrange equations corresponding to Toda-like systems [13]. An analogous reduction for the models with multicomponent "perfect" fluid was done earlier in [17, 18]. Earlier extentions of the Schwarzchild, Tangerlini, Reissner-Nordstrom and Majumdar-Papapetrou solutions to diverse dimensions see in [2, 3].

For cosmological models with antisymmetric forms without scalar fields any p-brane is equivalent to an anisotropic perfect fluid with the equations of state:

$$p_i = -\rho, \quad \text{or} \quad p_i = \rho, \tag{1.1}$$

when the manifold M_i belongs or does not belong to the brane worldvolume, respectively (here p_i is the pressure in M_i and ρ is the density, see Section 2).

In this paper we use this analogy in order to find a new family of exact spherically-symmetric solutions in the model with 1-component anisotropic fluid for more general equations of state (see Appendix for more familiar form of eqs. of state):

$$p_r = -\rho(2q-1)^{-1}, \qquad p_0 = \rho(2q-1)^{-1}, \tag{1.2}$$

and

$$p_i = \left(1 - \frac{2U_i}{d_i}\right)\rho/(2q-1), \tag{1.3}$$

$i > 1$, where ρ is a density, p_r is a radial pressure, p_i is a pressure in M_i, $i = 2, \ldots, n$. Here parameters U_i ($i > 1$) are arbitrary and the parameter $q > 0$ obey $q \neq 1/2$. The manifold M_0 is d_0-dimensiuonal sphere in our case and p_0 is the pressure in the tangent direction. The case $q = 1$ was considered earlier in [20].

The paper is organized as follows. In Section 2 the model is formulated. In Section 3 a subclass of spherically symmetric solutions (generalizing solutions from [20]) is presented and black hole solutions with integer q are singled out.

Section 4 deals with certain examples of solutions containing for $q = 1$ the Reissner-Nordström metric, $M2$ and $M5$ black brane metrics. In Section 5 the post-Newtonian parameters for the 4-dimensional section of the metric are calculated.

2 The model

Here, we consider a family of spherically symmetric solutions to Einstein equations with an anisotropic fluid matter source

$$R^M_N - \frac{1}{2}\delta^M_N R = kT^M_N \tag{2.1}$$

defined on the manifold

$$M = \underset{\substack{radial\\ variable}}{\mathbb{R}_{.}} \times \underset{\substack{spherical\\ variables}}{(M_0 = S^{d_0})} \times \underset{time}{(M_1 = \mathbb{R})} \times M_2 \times \ldots \times M_n, \tag{2.2}$$

with the block-diagonal metrics

$$ds^2 = e^{2\gamma(u)} du^2 + \sum_{i=0}^{n} e^{2X^i(u)} h^{(i)}_{m_i n_i} dy^{m_i} dy^{n_i}. \tag{2.3}$$

Here $\mathbb{R}_{.} = (a, b)$ is interval. The manifold M_i with the metric $h^{(i)}$, $i = 1, 2, \ldots, n$, is the Ricci-flat space of dimension d_i:

$$R_{m_i n_i}[h^{(i)}] = 0, \tag{2.4}$$

and $h^{(0)}$ is standard metric on the unit sphere S^{d_0}

$$R_{m_0 n_0}[h^{(0)}] = (d_0 - 1) h^{(0)}_{m_0 n_0}, \tag{2.5}$$

u is radial variable, κ is the multidimensional gravitational constant, $d_1 = 1$ and $h^{(1)} = -dt \otimes dt$.

The energy-momentum tensor is adopted in the following form

$$(T^M_N) = \mathrm{diag}(-(2q-1)^{-1}\rho, (2q-1)^{-1}\rho\delta^{m_0}_{k_0}, -\rho, p_2\delta^{m_2}_{k_2}, \ldots, p_n\delta^{m_n}_{k_n}), \tag{2.6}$$

where $q > 0$ and $q \neq 1/2$. The pressures p_i and the density ρ obeys the relations (1.3) with arbitrary constants U_i, $i > 1$.

In what follows we put $\kappa = 1$ for simplicity.

3 Exact solutions

Let us define

$$1^o. \quad U_0 = 0, \tag{3.1}$$

$$2^o. \quad U_1 = q, \tag{3.2}$$

$$3^o. \quad (U, U) = U_i G^{ij} U_j, \tag{3.3}$$

where $U = (U_i)$ is $(n+1)$-dimensional vector and

$$G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2-D} \tag{3.4}$$

are components of the matrix inverse to the matrix of the minisuperspace metric [15, 16]

$$(G_{ij}) = (d_i \delta_{ij} - d_i d_j) \tag{3.5}$$

and $D = 1 + \sum\limits_{i=0}^{n} d_i$ is the total dimension.

In our case the scalar product (3.3) reads

$$(U, U) = q^2 + \sum_{i=2}^{n} \frac{U_i^2}{d_i} + \frac{1}{2-D} \left(q + \sum_{i=2}^{n} U_i \right)^2 . \tag{3.6}$$

It is proved in Appendix that the relation 1^o implies $(U, U) > 0$.

For the equations of state (1.2) and (1.3) we have obtained the following spherically symmetric solutions to the Einstein equations (2.1) (see Appendix)

$$ds^2 = J_0 \left(\frac{dr^2}{1 - \frac{2\mu}{r^d}} + r^2 d\Omega^2_{d_0} \right) - J_1 \left(1 - \frac{2\mu}{r^d} \right) dt^2 + \sum_{i=2}^{n} J_i h^{(i)}_{m_i n_i} dy^{m_i} dy^{n_i}, \tag{3.7}$$

$$\rho = \frac{(2q-1)(dq)^2 P(P + 2\mu)(1 - 2\mu r^{-d})^{q-1}}{2(U, U) H^2 J_0 r^{2d_0}}, \tag{3.8}$$

by methods similar to obtaining p-brane solution [13]. Here $d = d_0 - 1$, $d\Omega^2_{d_0} = h^{(0)}_{m_0 n_0} dy^{m_0} dy^{n_0}$ is spherical element, the metric factors

$$J_i = H^{-2U^i/(U,U)}, \quad H = 1 + \frac{P}{2\mu}\left[1 - \left(1 - \frac{2\mu}{r^d}\right)^q\right]; \tag{3.9}$$

$P > 0$, $\mu > 0$ are constants and

$$U^i = G^{ij} U_j = \frac{U_i}{d_i} + \frac{1}{2-D}\sum_{j=0}^{n} U_j. \tag{3.10}$$

Using (3.10) and $U_0 = 0$ we get

$$U^0 = \frac{1}{2-D}\sum_{j=0}^{n} U_j \tag{3.11}$$

and hence one can rewrite (3.7) as follows

$$ds^2 = J_0 \left[\frac{dr^2}{1 - \frac{2\mu}{r^d}} + r^2 d\Omega^2_{d_0} - H^{-2q/(U,U)}\left(1 - \frac{2\mu}{r^d}\right) dt^2 + \right. \\ \left. + \sum_{i=2}^{n} H^{-2U_i/(d_i(U,U))} h^{(i)}_{m_i n_i} dy^{m_i} dy^{n_i}\right]. \tag{3.12}$$

Remark 1. We note that the density ρ is positive for $2q > 1$ and negative for $2q < 1$. For $2q = 1$ the solution also exists. In this case $\rho = 0$ and the energy-momentum tensor should be rewritten in terms of p_0

$$(T^M_N) = \mathrm{diag}(-p_0, p_0\delta^{m_0}_{k_0}, 0, p_2\delta^{m_2}_{k_2}, \ldots, p_n\delta^{m_n}_{k_n}), \tag{3.13}$$

where

$$p_0 = \frac{d^2 P(P+2\mu)(1-2\mu r^{-d})^{-1/2}}{8(U,U)H^2 J_0 r^{2d_0}}. \tag{3.14}$$

Black holes for natural q. For natural

$$q = 1, 2, \ldots \tag{3.15}$$

the metric has a horizon at $r^d = 2\mu = r_h^2$. Indeed, for these values of q the function $H(r) > 0$ is smooth in the interval $(r_*, +\infty)$ for some $r_* < r_h$. (For odd $q = 2m+1$ one get $r_* = 0$. A global structure of the black hole solution corresponding to these values of q will be a subject of a separate publication.

For $2U^0 \neq -1$ and $0 < q < 1$ we get a singularity $r^d \to 2\mu$. Indeed, due to Einstein equations the scalar curvature of the metric is proportional to

$$T_M^M = [D - 2 - 2\sum_{j=0}^{n} U_j] p_0 = (1 + 2U^0)(D-2) p_0 \tag{3.16}$$

but p_0 (proportional to ρ) diverges when $r^d \to 2\mu$ for $q < 1$ (see (3.8)).

Remark 2. For non-integer $q > 1$ the function $H(r)$ has a non-analitical behaviour in the vicinity of $r^d = 2\mu$. In this case one may conject that the limit $r^d \to 2\mu$ also corresponds to singularity but this subject needs a separate investigation.

4 Examples

Up till now U_i were arbitrary in our solution.

Here we consider certain examples of the solution with

$$U_2 = q d_2, \qquad U_i = 0, \tag{4.1}$$

for $i > 2$ and the equations of state: $p_2 = -\rho$, and $p_j = \rho/(2q-1)$ for $j > 2$. The $q = 1$ case describing the fluid analogue of p-brane solution with $p = d_2$ was considered in [20].

4.1 Solutions for $D = 4$

Let us consider the 4-dimensional space-time manifold $\mathbb{R} \times S^2 \times \mathbb{R}$. The metric and the density ¿from (3.12) and (3.8) read

$$ds^2 = H^{2/q} \left[\frac{dr^2}{1 - \frac{2\mu}{r}} + r^2 d\Omega_2^2 - H^{-4/q} \left(1 - \frac{2\mu}{r}\right) dt^2 \right], \tag{4.2}$$

$$\rho = \frac{(2q-1)P(P+2\mu)}{H^{2+(2/q)} r^4} \left(1 - \frac{2\mu}{r}\right)^{q-1}. \tag{4.3}$$

Here $H = 1 + \frac{P}{2\mu}\left[1 - \left(1 - \frac{2\mu}{r}\right)^q\right]$. For $q = 1$ by changing of variable $r = r' - P$ we obtain the standard Reissner-Nordström metric

$$ds^2_{RN} = -\left(1 - \frac{2GM}{r'} + \frac{Q^2}{r'^2}\right) dt^2 + \left(1 - \frac{2GM}{r'} + \frac{Q^2}{r'^2}\right)^{-1} dr'^2 + r'^2 d\Omega^2 \quad (4.4)$$

with the charge squared $Q^2 = P(P+2\mu)$ and the gravitational radius $GM = P + \mu$. Here, obviously, $Q^2 < (GM)^2$.

4.2 Solutions for $D = 11$

Here we consider two examples of solutions for the case $D = 11$ and $n = 3$. These solutions are generalizations of solutions for $q = 1$ from [20].

$(M2)_q$**-solutions**. For $U_2 = qd_2 = 2q$, $U_3 = 0$ we get from (3.12):

$$ds^2 = H^{1/(3q)} \left[\frac{dr^2}{1 - \frac{2\mu}{r^d}} + r^2 d\Omega^2_{d_0} - H^{-1/q} \left(1 - \frac{2\mu}{r^d}\right) dt^2 \right. \\ \left. + H^{-1/q} h^{(2)}_{m_2 n_2} dy^{m_2} dy^{n_2} + h^{(3)}_{m_3 n_3} dy^{m_3} dy^{n_3} \right]. \quad (4.5)$$

For $q = 1$ this formula gives the metric of the electric $M2$ black brane solution in 11-dimensional supergravity [8]. The density (3.8) has the following form:

$$\rho = \frac{(2q-1)d^2 P(P+2\mu)(1 - 2\mu r^{-d})^{q-1}}{4H^{2+(1/3q)} r^{2d_0}}. \quad (4.6)$$

$(M5)_q$**-solution**. Now we put $U_2 = qd_2 = 5q$. The metric reads:

$$ds^2 = H^{2/(3q)} \left[\frac{dr^2}{1 - \frac{2\mu}{r^d}} + r^2 d\Omega^2_{d_0} - H^{-1/q} \left(1 - \frac{2\mu}{r^d}\right) dt^2 + \right. \\ \left. H^{-1/q} h^{(2)}_{m_2 n_2} dy^{m_2} dy^{n_2} + h^{(3)}_{m_3 n_3} dy^{m_3} dy^{n_3} \right], \quad (4.7)$$

and the density is

$$\rho = \frac{(2q-1)d^2 P(P+2\mu)(1 - 2\mu r^{-d})^{q-1}}{4H^{2+(2/3q)} r^{2d_0}}. \quad (4.8)$$

For $q = 1$ we get the metric of the magnetic $M5$ black brane solution in 11-dimensional supergravity [8].

5 Physical parameters

5.1 Gravitational mass and PPN parameters

Here we put $d_0 = 2$ ($d = 1$). Let us consider the 4-dimensional space-time section of the metric (3.12). Introducing a new radial variable by the relation:

$$r = R\left(1 + \frac{\mu}{2R}\right)^2, \tag{5.1}$$

we rewrite the 4-section in the following form:

$$ds^2_{(4)} = H^{-2U^0/(U,U)}\left[-H^{-2q/(U,U)}\left(\frac{1-\frac{\mu}{2R}}{1+\frac{\mu}{2R}}\right)^2 dt^2 + \left(1+\frac{\mu}{2R}\right)^4 \delta_{ij}dx^i dx^j\right] \tag{5.2}$$

$i, j = 1, 2, 3$. Here $R^2 = \delta_{ij}x^i x^j$.

The parametrized post-Newtonian (Eddington) parameters are defined by the well-known relations

$$g^{(4)}_{00} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{5.3}$$
$$g^{(4)}_{ij} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{5.4}$$

$i, j = 1, 2, 3$. Here

$$V = \frac{GM}{R} \tag{5.5}$$

is the Newtonian potential, M is the gravitational mass and G is the gravitational constant.

¿From (5.2)-(5.4) we obtain:

$$GM = \mu + \frac{Pq(q + U^0)}{(U,U)} \tag{5.6}$$

and

$$\beta - 1 = \frac{|A|}{(GM)^2}(q + U^0), \tag{5.7}$$
$$\gamma - 1 = -\frac{Pq}{(U,U)GM}(q + 2U^0), \tag{5.8}$$

where

$|A| = \frac{1}{2}q^2 P(P+2\mu)/(U,U)$ (see Appendix), or, equivalently,

$P = -\mu + \sqrt{\mu^2 + 2|A|(U,U)q^{-2}} > 0$.

For fixed U_i the parameter $\beta - 1$ is proportional to the ratio of two physical parameters: the anisotropic fluid density parameter $|A|$ and the gravitational radius squared $(GM)^2$. For compact internal spaces the parameter $|A|$ is proportional to the effective mass of the fluid outside the external horizon (for natural q), i.e. to the integral of ρ over the region $r^d > 2\mu$.

5.2 Hawking temperature

The Hawking temperature of the black hole may be calculated using the well-known relation [19]

$$T_H = \frac{1}{4\pi\sqrt{-g_{tt}g_{rr}}} \frac{d(-g_{tt})}{dr}\bigg|_{horizon}. \tag{5.9}$$

We get

$$T_H = \frac{d}{4\pi(2\mu)^{1/d}} \left(1 + \frac{P}{2\mu}\right)^{-q/(U,U)}. \tag{5.10}$$

Here $q = 1, 2, \ldots$.

For the 4-dimensional solution (4.2) we get $T_H = \frac{1}{8\pi\mu}\left(1 + \frac{P}{2\mu}\right)^{-2/q}$. For $D = 11$ metrics (4.5) and (4.7) the Hawking temperature reads

$T_H = \frac{d}{4\pi(2\mu)^{1/d}}\left(1 + \frac{P}{2\mu}\right)^{-1/(2q)}$.

6 Conclusions

In this paper, using our methods developed earlier for obtaining perfect fluid and p-brane solutions, we have considered a family of spherically symmetric solutions in the model with 1-component anisotropic fluid when the equations

of state (1.2) and (1.3) are imposed. The metric of any solution contains $(n-1)$ Ricci-flat "internal" space metrics and depends upon arbitrary parameters U_i, $i > 1$.

For $q = 1$ and certain equations of state (with $p_i = \pm\rho$) the metric of the solution coincides with that of black brane (or black hole) solution in the model with antisymmetric forms without dilatons [20]. For natural numbers $q = 1, 2, \ldots$ we obtained a family of black hole solutions.

Here we also considered certain examples of solutions with horizon, e.g. (fluid) generalizations of charged black hole and $M2$, $M5$ black brane solutions.

We have also calculated for possible estimations of observable efffects of extra dimensions the post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric and the Hawking temperature as well. The parameter $\beta - 1$ is written in terms of ratios of the physical parameters: the perfect fluid parameter $|A|$ and the gravitational radius squared $(GM)^2$.

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Appendix

A Lagrange representation

It is more convenient for finding of exact solutions, to write the stress-energy tensor in cosmological-type form

$$(T_N^M) = \mathrm{diag}(-\hat{\rho}, \hat{p}_0 \delta_{k_0}^{m_0}, \hat{p}_1 \delta_{k_1}^{m_1}, \dots, \hat{p}_n \delta_{k_n}^{m_n}), \tag{A.1}$$

where $\hat{\rho}$ and $\hat{p}_i$ are "effective" density and pressures, respectively, depending upon the radial variable u and the physical density ρ and pressures p_i are related to the effective ("hat") ones by formulas

$$\rho = -\hat{p}_1, \quad p_r = -\hat{\rho}, \quad p_i = \hat{p}_i, \quad (i \neq 1). \tag{A.2}$$

The equations of state may be written in the following form

$$\hat{p}_i = \left(1 - \frac{2U_i}{d_i}\right) \hat{\rho}, \tag{A.3}$$

where U_i are constants, $i = 0, 1, 2, \dots, n$. It follows from (A.2), (A.3) and $U_1 = q$ that

$$\rho = (2q - 1)\hat{\rho}. \tag{A.4}$$

The conservation law equations $\nabla_M T_N^M = 0$ (following from Einstein equations) may be written, due to relations (2.3) and (A.1) in the following form:

$$\dot{\hat{\rho}} + \sum_{i=0}^{n} d_i \dot{X}^i (\hat{\rho} + \hat{p}_i) = 0. \tag{A.5}$$

Using the equation of state (A.3) we get

$$\hat{\rho} = -A e^{2U_i X^i - 2\gamma_0}, \tag{A.6}$$

where $\gamma_0(X) = \sum\limits_{i=0}^{n} d_i X^i$ and A is constant.

The Einstein equations (2.1) with the relations (A.3) and (A.6) imposed are equivalent to the Lagrange equations for the Lagrangian

$$L = \frac{1}{2} e^{-\gamma + \gamma_0(X)} G_{ij} \dot{X}^i \dot{X}^j - e^{\gamma - \gamma_0(X)} V, \tag{A.7}$$

where

$$V = \frac{1}{2} d_0 (d_0 - 1) e^{2U_i^{(0)} X^i} + A e^{2U_i X^i} \tag{A.8}$$

is the potential and the components of the minisupermetric G_{ij} are defined in (3.5).

$$U_i^{(0)} X^i = -X^0 + \gamma_0(X), \qquad U_i^{(0)} = -\delta_i^0 + d_i, \qquad A_0 = \frac{1}{2} d_0 (d_0 - 1), \tag{A.9}$$

$i = 0, \ldots, n$ (for cosmological case see [17, 18]).

For $\gamma = \gamma_0(X)$, i.e. when the harmonic time gauge is considered, we get the set of Lagrange equations for the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{X}^i \dot{X}^j - V, \tag{A.10}$$

with the zero-energy constraint imposed

$$E = \frac{1}{2} G_{ij} \dot{X}^i \dot{X}^j + V = 0. \tag{A.11}$$

It follows from the restriction $U_0 = 0$ that

$$(U^{(0)}, U) \equiv U_i^{(0)} G^{ij} U_j = 0. \tag{A.12}$$

Indeed, the contravariant components $U^{(0)i} = G^{ij} U^{(0)} j$ are the following ones

$$U^{(0)i} = -\frac{\delta_0^i}{d_0}. \tag{A.13}$$

Then we get $(U^{(0)}, U) = U^{(0)i} U_i = -U_0/d_0 = 0$. In what follows we also use the formula

$$(U^{(0)}, U^{(0)}) = \frac{1}{d_0} - 1 < 0, \tag{A.14}$$

for $d_0 > 1$.

Now we prove that $(U, U) > 0$. Indeed, minisupermetric has the signature $(-, +, \ldots, +)$ [15, 16], vector $U^{(0)}$ is time-like and orthogonal to vector $U \neq 0$. Hence the vector U is space-like.

B General spherically symmetric solutions

When the orthogonality relations (A.12) and 3^o of (3.1) are satisfied the Euler-Lagrange equations for the Lagrangian (A.10) with the potential (A.8) have the following solutions (see relations from [18] adopted for our case):

$$X^i(u) = -\sum_{\alpha=0}^{1} \frac{U^{(\alpha)i}}{(U^{(\alpha)}, U^{(\alpha)})} \ln |f_\alpha(u - u_\alpha)| + c^i u + \bar{c}^i, \tag{B.15}$$

where $U^{(1)} = U$, u_α are integration constants; and vectors $c = (c^i)$ and $\bar{c} = (\bar{c}^i)$ are orthogonal to the $U^{(\alpha)} = (U^{(\alpha)i})$, i.e. they satisfy the linear constraint relations

$$U^{(0)}(c) = U_i^{(0)} c^i = -c^0 + \sum_{j=0}^{n} d_j c^j = 0, \tag{B.16}$$

$$U^{(0)}(\bar{c}) = U_i^{(0)} \bar{c}^i = -\bar{c}^0 + \sum_{j=0}^{n} d_j \bar{c}^j = 0, \tag{B.17}$$

$$U(c) = U_i c^i = 0, \tag{B.18}$$

$$U(\bar{c}) = U_i \bar{c}^i = 0. \tag{B.19}$$

Here

$$f_\alpha(\tau) = \begin{array}{ll} R_\alpha \dfrac{\sinh(\sqrt{C_\alpha}\tau)}{\sqrt{C_\alpha}}, & C_\alpha > 0, \quad \eta_\alpha = +1, \\ R_\alpha \dfrac{\cosh(\sqrt{C_\alpha}\tau)}{\sqrt{C_\alpha}}, & C_\alpha > 0, \quad \eta_\alpha = -1, \\ R_\alpha \dfrac{\sin(\sqrt{|C_\alpha|}\tau)}{\sqrt{|C_\alpha|}}, & C_\alpha < 0, \quad \eta_\alpha = +1, \\ & \\ R_\alpha \tau, & C_\alpha = 0, \quad \eta_\alpha = +1, \end{array} \tag{B.20}$$

$\alpha = 0, 1$; where $R_0 = d_0 - 1$, $\eta_0 = 1$, $R_1 = \sqrt{2|A|(U,U)}$, $\eta_1 = -\mathrm{sign} A$.

The zero-energy constraint, corresponding to the solution (B.15) reads

$$E = \frac{1}{2} \sum_{\alpha=0}^{1} \frac{C_\alpha}{(U^{(\alpha)}, U^{(\alpha)})} + \frac{1}{2} G_{ij} c^i c^j = 0. \tag{B.21}$$

Special solutions. The horizon condition (i.e. infinite time of propagation of light for $u \to +\infty$) lead us to the following integration constants

$$\bar{c}^i = 0, \tag{B.22}$$

$$c^i = \bar{\mu} \sum_{\alpha=0}^{1} \frac{U_1^{(\alpha)} U^{(\alpha)i}}{(U^{(\alpha)}, U^{(\alpha)})} - \bar{\mu}\delta_1^i, \tag{B.23}$$

$$C_\alpha = (U_1^{(\alpha)})^2 \bar{\mu}^2, \tag{B.24}$$

where $\bar{\mu} > 0$, $\alpha = 0, 1$.

We also introduce a new radial variable $r = r(u)$ by relations

$$\exp(-2\bar{\mu}u) = 1 - \frac{2\mu}{r^d}, \quad \mu = \bar{\mu}/d > 0, \quad d = d_0 - 1, \tag{B.25}$$

and put $u_1 < 0$, $A < 0$, $u_0 = 0$.

The relations of the Appendix imply the formulae (3.7) and (3.8) for the solution from Section 3 with

$$H = \exp(-\bar{\mu}U_1 u) f_1(u - u_1), \qquad A = -\frac{(dq)^2}{2(U, U)} P(P + 2\mu), \tag{B.26}$$

$P > 0$.

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